# Nonlinear Analysis for Natural Frequencies of Rectangular and NACA0012 Aerofoil Cross-section Beam

Bahadursha A V L Sainadh<sup>1</sup>, V Mani Kumar<sup>2</sup> M Tech student<sup>1</sup>, Assistant Professor<sup>2</sup>, Department of Mechanical Engineering<sup>1,2</sup>, JNTUK-UCEV<sup>1,2</sup> Email: <u>sainadhbahadursha@gmail.com<sup>1</sup></u>, <u>velugula.mani@gmail.com</u><sup>2</sup>

Abstract-Natural frequencies are responsible for failures of the aircraft wing due to resonance phenomenon caused by external dynamic loading by atmosphere. Rectangular cross-sectional beam's natural frequencies are not suitable to use for aircraft wing with aerofoil cross section. So aerofoil cross-sectional shape data derived for NACA0012 symmetrical aerofoil. Nonlinear analysis was carried out by using nonlinear 3-D Euler Bernoulli beam theory and extended Hamilton principle for both rectangle and NACA0012 aerofoil cross sections. Method of separation of variable procedure followed to get solutions of equations of motion of beams and its corresponding in-plane and out-of-plane natural frequencies.

### Index Terms-Natural frequencies; NACA0012; aerofoil; Euler Bernoulli beam theory; Hamilton principle;

in-plane; out-of-plane

### 1. INTRODUCTION

Generally rectangular cross-sectional beams can be analysed for equations of motion of beams, boundary conditions and natural frequencies by different theories like Euler Bernoulli beam theory, shear deformable beam theory and three dimensional beam theory. But rectangular cross-sectional beam's natural frequencies are not suitable to use for aircraft wing with aerofoil cross section. Inaccuracy in natural frequencies may lead to failure of the beam by resonance. Different types of aerofoils were used for aircraft wings according to the requirement of lift and drag forces. NACA0012 is one of the symmetrical aerofoil. It has mean camber line and thickness distribution equations to get aerofoil surfaces in equation format which helps to derive its geometrical data.

Crespa de silva and Glynn<sup>[1, 2]</sup> formulated a set of mathematically consistent governing equations of motion describing the non-planar, nonlinear dynamics of an inextensional and extensional-flexural-flexuraltorsional beams. The beam is assumed to undergo flexure about two principal axes and torsion. Nonlinear curvature and nonlinear inertia effects were also considered. They<sup>[3, 4]</sup> also analyzed the nonplanar, nonlinear forced oscillations of a fixed-free beam by a perturbation technique with the objective of determining quantitative and qualitative information about the frequency response. Nayfeh and Pai<sup>[5]</sup> studied nonlinear analysis on different types of structures like strings, bars, beams, shells and plates. In the present work, firstly NACA0012 aerofoil geometrical data was derived by using Prandtl elastic membrane theory and energy method. Then nonlinear analysis was carried out by using nonlinear 3-D Euler Bernoulli beam theory and extended Hamilton principle for both rectangle and NACA0012 aerofoil cross section. Method of separation of variable procedure followed to get solutions of equations of motion of beams and its corresponding in-plane and out-of-plane natural frequencies.

Prime'' indicates the partial derivative with respect to space coordinate's' and dot '.' indicates the partial derivative with respect to time. u, v, w are the displacements along x,y,z axes respectively.  $\phi$  is the torsional displacement about x axis.

### 2. CROSS-SECTIONAL DATA

The material used is aluminium alloy with density 2730 kg/m<sup>3</sup>, Young's modulus 69 Gpa, Poisson's ratio 0.33.

### 2.1. Rectangle shape data

Table 1. Rectangle shape data.<sup>[7]</sup>

Length (L)	0.08 m
Breadth (B)	0.0127 m
Height (H)	0.0004064 m
Area (A)	5.16128 x 10 <sup>-6</sup> m <sup>2</sup>
Polar area moment of inertia about x axis $(I_1)$	6.944293 x 10 <sup>-11</sup> m <sup>4</sup>
Area moment of inertia about y axis $(I_2)$	6.9371904 x 10 <sup>-11</sup> m <sup>4</sup>
Area moment of inertia about z axis $(I_3)$	7.103682 x 10 <sup>-14</sup> m <sup>4</sup>

Bending rigidity about y axis (D <sub>2</sub> )	4.786661 N-m <sup>2</sup>
Bending rigidity about z axis (D <sub>3</sub> )	0.004901 N-m <sup>2</sup>
Torsional rigidity about x axis $(D_1)$	$7.222083 \times 10^{-3} \mathrm{N} \cdot \mathrm{m}^2$



Fig. 1. Rectangular cross-section

Above moment of inertias and rigidities can be calculated from general inertia and rigidity formulae for rectangle.<sup>[8,9]</sup> The mass moment of inertia  $(j_i)$  can be calculated by multiplying area moment of inertias with density of the material.

#### 2.2. NACA0012 aerofoil shape data

Symmetrical aerofoil NACA0012 has zero camber and 12% of chord length as section thickness. Enlarge the chord length of aerofoil from c=1 to c=c and use the actual thickness distribution equation<sup>[10]</sup> to get the thickness distribution equation as follow.

$$y = \pm (5cXX) \left[ A_0 z^{\binom{1}{2}} + A_1 z + A_2 z^2 + A_3 z^3 + A_4 z^4 \right] ..(1)$$
  
Where  $A_0 = \frac{a_0}{c^{\binom{1}{2}}} = \frac{0.2969}{0.0127^{\binom{1}{2}}} = 2.634561$ ,  
 $A_1 = \frac{a_1}{c} = \frac{-0.1260}{0.0127} = -9.921259$ ,  
 $A_2 = \frac{a_2}{c^2} = \frac{-0.3516}{0.0127^2} = 2179.92436$   
 $A_3 = \frac{a_3}{c^3} = \frac{0.2843}{0.0127^3} = 138792.4036$   
 $A_4 = \frac{a_4}{c^4} = \frac{-0.1036}{0.0127^4} = -3982399.93$  (Consider closed

trailing edge)



Fig.2. Symmetrical NACA0012 aerofoil with coordinate axis at its centroid

Area and centroid are found from equation (1). The coordinate axis was shifted from the leading edge to the centroid of the aerofoil to avoid the shear centre and non-symmetrical bending effect. Then the equation changes as follow.

$$Y = \pm (5cXX) \begin{bmatrix} A_0 (Z + \overline{z})^{\binom{1}{2}} + A_1 (Z + \overline{z}) \\ + A_2 (Z + \overline{z})^2 + A_3 (Z + \overline{z})^3 + A_4 (Z + \overline{z})^4 \end{bmatrix}$$
...(2)

By using equation 2, get the area moment of inertias by integrating over  $Z=-\overline{z}$  to  $Z=c-\overline{z}$ . Bending rigidities can be found by multiplying the area moment of inertias with corresponding young's modulus. But torsional rigidity of aerofoil must be found by using the Prandtl elastic membrane function  $(\phi)$  and energy method.<sup>[8,9]</sup>

$$D_{1i} = \left[ \frac{\left(\frac{8}{9}a^{3}G\left(\int_{Z=-\bar{z}}^{Z=c-\bar{z}}\psi^{3} dZ\right)^{2}\right)}{\left(a^{2}\int_{Z=-\bar{z}}^{Z=c-\bar{z}}\psi^{3}(\psi')^{2} dZ\right) + \left(\int_{Z=-\bar{z}}^{Z=c-\bar{z}}\psi^{3} dZ\right)} \right] ...(3)$$

Equation 3 is the final reduced torsional rigidity of the NACA0012 aerofoil where G is shear modulus, a = 5 x c x t, t is maximum section thickness and

$$\psi = \begin{bmatrix} A_0 \left( Z + \overline{z} \right)^{\binom{1}{2}} + A_1 \left( Z + \overline{z} \right) \\ + A_2 \left( Z + \overline{z} \right)^2 + A_3 \left( Z + \overline{z} \right)^3 + A_4 \left( Z + \overline{z} \right)^4 \end{bmatrix}$$

$$..(4)$$

By using all the above equation, the following table can be obtained.

Table 2. NACA0012 aerofoil shape data.<sup>[7]</sup>

Chord length (c)	0.0127 m
Centroid $(\overline{z},\overline{y})$	(5.307184 x 10 <sup>-3</sup> , 0) m

Area (A)	1.317836 x 10 <sup>-5</sup> m <sup>2</sup>
Polar area moment of inertia about x axis $(I_1)$	$1.1732 \ge 10^{-10} \text{ m}^4$
Area moment of inertia about y axis $(I_2)$	$1.1555 \ge 10^{-10} \text{ m}^4$
Area moment of inertia about z axis $(I_3)$	$1.763 \ge 10^{-12} \text{ m}^4$
Bending rigidity about y axis (D <sub>2</sub> )	7.9732 N-m <sup>2</sup>
Bending rigidity about z axis (D <sub>3</sub> )	0.1216 N-m <sup>2</sup>
Torsional rigidity about x axis (D <sub>1</sub> )	$0.0605 \mathrm{N}\mathrm{-m}^2$

### 3. THEORETICAL GOVERNING EQUATIONS AND ITS SOLUTION

The theoretical governing equations applicable for NACA0012 aerofoil were derived using the following steps.

# Steps to derive the governing equations and natural frequencies

- Transformations using two Euler angles
- Taylor series expansion of transformations using Euler angles
- Nonlinear 3D Euler Bernoulli beam theory
- Extended Hamilton Principle
- Finding equations of motion of beam of third order
- Converting into first order equations of motion of beam
- Apply method of separation of variables in partial differential equations obtained
- Use the boundary conditions to get natural frequencies

Transformations using two Euler angles and Taylor series expansions are taken from the Nayfeh and pai text book on linear and nonlinear structural mechanics.<sup>[5]</sup>

# **3.1.** Modified Extended Hamilton principle by using nonlinear 3D Euler Bernoulli beam theory

For a structure, according to extended Hamilton principle

$$\int_{t_1}^{t_2} (\delta T \text{-} \delta \Pi \text{+} \delta W_{nc}) dt = 0$$

..(5)

Where  $\delta T$  is the kinetic energy variation for a infinitesimal part of the beam,  $\delta \Pi$  is the elastic energy variation for a infinitesimal part of the beam,  $\delta W_{nc}$  is the external energy variation for a infinitesimal part of the beam.<sup>[5]</sup>

$$\begin{bmatrix} \int_{0}^{t} \int_{0}^{L_{i}} \left\{ -F_{1i} \left( T_{11i} \delta u_{i}' + T_{12i} \delta v_{i}' + T_{13i} \delta w_{i}' \right) \\ -\left( A_{ui} + c_{1i} \dot{u}_{i} - q_{1i} \right) \delta u_{i} \\ -\left( A_{vi} + c_{2i} \dot{v}_{i} - q_{2i} \right) \delta v_{i} \\ -\left( A_{wi} + c_{3i} \dot{w}_{i} - q_{3i} \right) \delta w_{i} \\ +\left( M_{1i}' + M_{3i} \rho_{2i} - M_{2i} \rho_{3i} - A_{\theta_{1i}} - c_{4} \dot{\phi}_{i} + q_{4i} \right) \delta \theta_{1i} \\ +\left( M_{2i}' - M_{3i} \rho_{1i} + M_{1i} \rho_{3i} - A_{\theta_{2i}} \right) \delta \theta_{2i} \\ +\left( M_{3i}' + M_{2i} \rho_{1i} - M_{1i} \rho_{2i} - A_{\theta_{3i}} \right) \delta \theta_{3i} \right\} ds dt \\ + \int_{0}^{t} \left[ M_{1i} \delta \theta_{1i} + M_{2i} \delta \theta_{2i} + M_{3i} \delta \theta_{3i} \right]_{0}^{L_{i}} dt \qquad \dots (6)$$

The above Hamilton equation was used to get the equations of motion of the beam. Firstly on solving, third order equations of motion can be obtained. Reduce the third order equations to first order and divide by  $m = \rho A$  to get following first order equations of motions and boundary conditions.

$$\begin{aligned} \ddot{u}_{i} - \frac{E}{\rho} u_{i}''=0 \\ \ddot{v}_{i} + \frac{EI_{3}}{\rho A} v_{i}''' - \frac{I_{3}}{A} \ddot{v}_{i}''=0 \\ \ddot{w}_{i} + \frac{EI_{2}}{\rho A} w_{i}''' - \frac{I_{2}}{A} \ddot{w}_{i}''=0 \\ \ddot{\phi}_{i} - \frac{D_{11}}{\rho I_{1}} \phi_{i}''=0 \\ ...(7) \end{aligned}$$

Boundary conditions are

$$\begin{split} & u_{i}(0,t)=0, v_{i}(0,t)=0, w_{i}(0,t)=0; \\ & \phi_{i}(0,t)=0; w_{i}'(0,t)=0, v_{i}'(0,t)=0 \\ & u_{i}'(L_{i},t)=0, v_{i}'''(L_{i},t)-\frac{\rho}{E} \ddot{v}_{i}'(L_{i},t)=0, \\ & w_{i}'''(L_{i},t)-\frac{\rho}{E} \ddot{w}_{i}'(L_{i},t)=0; \\ & \phi_{i}'(L_{i},t)=0; v_{i}''(L_{i},t)=0, v_{i}''(L_{i},t)=0 \end{split}$$

#### 3.2. Method of separation of variables

Above equations of motions are assumed to be separable in space  $(s_i)$  and time (t). Also assuming the beams vibrate periodic in nature as in simple harmonic motion with respect time.

$$u_i = U_i(s_i)q_i^u(t);$$
  
 $v_i = V_i(s_i)q_i^v(t);$ 

..(9)

$$w_{i} = W_{i}(s_{i})q_{i}^{w}(t);$$
  
$$\phi_{i} = \Phi_{i}(s_{i})q_{i}^{\phi}(t);$$

Equation of motion changes as-

$$U_{i}''(s_{i}) + \frac{\rho\omega_{in}^{2}}{E} U_{i}(s_{i}) = 0$$

$$V_{i}''''(s_{i}) + \frac{\rho\omega_{in}^{2}}{E} V_{i}''(s_{i}) - \frac{\rho A \omega_{in}^{2}}{EI_{3}} V_{i}(s_{i}) = 0$$

$$W_{i}''''(s_{i}) + \frac{\rho\omega_{out}^{2}}{E} W_{i}''(s_{i}) - \frac{\rho A \omega_{out}^{2}}{EI_{3}} W_{i}(s_{i}) = 0$$

$$\Phi_{i}''(s_{i}) + \frac{\rho I_{1} \omega_{out}^{2}}{D_{11}} \Phi_{i}(s_{i}) = 0$$
...(10)

# **3.3.** In-plane and out-of-plane motion boundary conditions of the beam

In-plane motions primary beam

$$\begin{split} U_1(0) = 0, \ U_1'(L_1) = 0; \\ V_1(0) = 0, \ V_1'(0) = 0, \ V_1''(L_1) = 0, \ V_1'''(L_1) - \frac{\rho \omega_{in}^2}{E} \ V_1'(L_1) = 0; \\ ..(11) \end{split}$$

Out-of-plane motions primary beam

$$\Phi_{1}(0)=0, \Phi_{1}'(L_{1})=0;$$

$$W_{1}(0)=0, W_{1}'(0)=0, W_{1}''(L_{1})=0,$$

$$W_{1}'''(L_{1})-\frac{\rho\omega_{out}^{2}}{E}W_{1}'(L_{1})=0;$$
...(12)

Using the equations (5-12), solve partial differential equations by substituting the cross-sectional shape data to get only unknown variable i.e., in-plane and out-of-plane natural frequencies of the beam.

#### 4. RESULTS AND CONCLUSION

The calculated results from the above equation are accumulated in following table. Obtained theoretical results are compared between two cross-sections and validated using ANSYS results taken from the journal entitled as "Modal analysis of multi-segmented beam with rectangular and naca0012 aerofoil cross-section in ANSYS"<sup>[11]</sup>



Fig.3. Natural frequencies of rectangular beam



Fig.4. Natural frequencies of NACA0012 aerofoil beam

From the above two graphs, it is clear that rectangle and aerofoil cross-section with same width has large difference in natural frequencies. Theoretical and ANSYS results are well in agreement. So aerofoil shape data derived from the Prandtl elastic membrane theory and energy method should be used in determining natural frequencies of the beam. The sixth modal natural frequency of NACA0012 aerofoil beam was very large natural frequency as it is caused by torsional displacement  $\phi$  with high torsional rigidity compared to rectangle cross-section.

### 5. FUTURE SCOPE

Multi-segmented beams can be dealt to get natural frequencies morphing wing aircraft with aerofoil cross-section. Unsymmetrical aerofoil like NACA2412 can be used to improve the aerofoil resisting capacity to resonance. Third order equation of motions can be taken to get more accurate results. Forced frequency response of Z-shaped beam can be solved by using Nayfeh and Pai dynamic response of beam analysis. Cross section with irregular shear centre cases can be dealt to get the equation of motion for complicated cross sections.

### REFERENCES

- Crespo da Silva; Glynn: Nonlinear flexuralflexural-torsional dynamics of the inextensional beams. 1. Equations of motion, Journal of Structural Mechanics.
- [2] Crespo da Silva; Glynn: Nonlinear flexuralflexural-torsional dynamics of the extensional beams. 1. Equations of motion, Journal of Structural Mechanics.
- [3] Crespo da Silva; Glynn: Nonlinear flexuralflexural-torsional dynamics of the inextensional beams. 2. Forced motions, Journal of Structural Mechanics.
- [4] Crespo da Silva; Glynn: Nonlinear flexuralflexural-torsional dynamics of the extensional beams. 2. Forced motions, Journal of Structural Mechanics.
- [5] Ali H. Nayfeh; P. Frank Pai: Linear and nonlinear structural mechanics, A text book published by Wiley Publications.
- [6] K. Y. Narasimhan: Torsion of an elastic cylinder of unsymmetrical aerofoil cross section, Journal of AIAA, 1965.
- [7] W. Zhang; W. H. Hu; D.X.Cao; M.H.Yao: Vibration frequencies and modes of a z-shaped beam with variable folding angles, ASME journal.
- [8] Arthur P. Boresi; Richard J. Schmidt: Advanced mechanics of materials, a textbook published by Wiley Publications.
- [9] S. P. Timoshenko; J. N. Goodier: Theory of elasticity, a textbook published by Mc Graw Hill.
- [10] Aerofoil dat files from <u>www.airfoiltools.com</u>,
- [11] Bahadursha A V L Sainadh; V Mani Kumar: Modal analysis of multi-segmented beam with rectangular and NACA0012 aerofoil cross-section in ANSYS, journal published in JETIR, October 2018, Volume 5, Issue 10, pg 99-103.